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Notes on Optimal Growth, Climate Change Calamities, Adaptation and Mitigation

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Abstract*

A strategy of inclusion of adaptation and mitigation expenses in a model of optimal growth under threat of climate change calamities is discussed in these exploratory notes. Calamity is the result of a shock that reduces the utility level (even to extinction forever) and/or triggers a fundamental change of the economic structure. Mitigation expenses reduce the long-run probability of a calamity or the speed of convergence to it; adaptation expenses help to improve the standard of living after the calamity. The willingness to contribute to those expenses and the effects on the long-run capital stock of the economy depend on perceptions on how they will modify the law of evolution of probabilities of the shock and the standard of living after the shock. The choice between a clean technology and one that increases GHG emissions is also discussed.

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1. Introduction

In these exploratory notes we shall discuss a strategy for the inclusion of adaptation and mitigation expenses in a problem of optimal growth for a representative agent under the threat of climate change calamities. Calamity is defined as a probabilistic abrupt shock that negatively influences negatively the utility level and/or triggers a fundamental change of the economic structure. It has the potential of being a case of "extinction forever"[1] and is a non-insurable event for the representative agent, who must undertake self-protection policies.

We shall evaluate the impact of mitigation and adaptation expenses on the probability of events, on the long-run capital stock and consequently on the standard of living. We shall see that the perception and the interpretation of the (scientific) information by the society (the representative agent) will influence willingness to contribute to expenses.

A first fact to take into account is that the analysis of effects of climate change, and even of possible remedial policies, has to be conducted under uncertainty.[2] Ambiguities in information and the complexity of the dynamic multidimensional phenomena leave small room for certainty.

To model the law of evolution of probabilities, we shall borrow the framework presented by Kamien and Schwartz in a sequel of papers on industrial organization and operational analysis; see Kamien and Schwartz (1971a and 1971b). They considered the case of an incumbent who conjectures that the price charged for a product will influence the probability of entry; this probability is assumed to evolve following the dynamics governed by a hazard rate. Their approach was inspired by the theory of reliability of mechanical devices.

In the case of our paper, given scientific uncertainty, that hazard rate will be reinterpreted as the speed of growth of the conditional probability of occurrence of a

[1] Pessimistic points of view are of course challenged; see, for example, Mendelsohn (2009). The evidence that supports the pessimistic view is also under review, see *The Economist*, July 5, July 2010. A more optimistic position would reduce the problem of climate change to the standard treatment of environmental economics, putting its effects at the level of discomfort in the utility function, or as a loss of productivity.

[2] See Pyndick (2009) and Weitzman (2009). The classic paper of Ulph and Ulph (1997) studies the case of mitigation expenses to address global warming under irreversibility and learning, but they do not consider the possibility of influencing the stochastic process; see also Ingham, Ma and Ulph (2005a) and (2005b) for interesting discussions.

calamity. But the role of price will be given to the production level or the mitigation expenses.

We shall also consider a different case in which it is the long-run residual probability that can be modified by policies of mitigation and production decisions. However, at variance with Kamien and Schwartz,[3] the residual (long-run) probability of the calamity will not necessarily be assumed to be one, as in the case of a mechanical device that eventually fail. Instead, we may conceive of a scenario in which Nature is a resilient entity and our present standard of living is not inexorably doomed; this is important for the allocation of resources because the belief that the doomsday is unavoidable may favor the reduction of expenses in mitigation.

There has been previous work using this setting. Examples of applications of reliability theory to the case of environmental issues can be found in Tsur and Zemel (2006a) and (2006b).

Chisari and Kessides (2009) use a similar framework to discuss a model of regulatory threat, when a firm estimates the probability of being fined or regulated, and thinks that it can influence those probabilities with its pricing behavior. As mentioned above, the problem is discussed in operational research for determining the replacement of machines with uncertain failure. Kamien and Schwartz (1971b) gave necessary conditions for an optimal replacement time for machineries. More recently, Dogramaci and Sethi (2007) discussed a model of machinery replacement in which the decision of when replacing a machine is endogenous, and quoted a result (to be published) that shows that Kamien and Schwartz conditions are also sufficient.

There are also several points of contact of this presentation with the existing literature on optimal exploitation of natural resources. In natural resources economics it is habitual to represent the available initial stock with an isoperimetric constraint, defined by the initial endowment. For example, in his classical paper, Koopmans (1974) constructs a model of optimal exploitation of a natural resource which is indispensable for life. He assumes a minimum consumption level and shows that an increase in the rate of discount

[3] In their model, once entrants get into the market, they will stay in it forever and the incumbent must anticipate the rules of coexistence with them and the expected profit.

advances the doomsday.[4] Loury (1978) analyzes the case of unknown size of the reserves of a natural resource; he obtains an optimality condition that gives the trade-off between living one more instant (and enjoying a discounted flow of utilities from the future) or enjoying one more unit of consumption in the present.

From the perspective of environmental economics, we know that the externalities of pollution can be included in the utility function or at the level of the production function. Normally, they are modeled as externalities from one set of agents onto others. As in normative economics, those externalities have to be addressed through market-based incentives, command-and-control instruments, or some reallocation of property rights, at least in the traditional strategy.

In the case of this paper, we will be discussing the special case of abrupt and irreversible changes of climate that could negatively influence the living standard of a community, represented by a single agent. This representative agent has to compute the optimal growth solution for the whole society, and therefore externalities are fully internalized.[5] However, his control of the program is imperfect, since the state of the environment alters the probability of a calamity. Consequently, the representative agent has to take into account not only the accumulation of capital, but also the accumulation of probabilities of facing an impact that could reduce welfare in a dramatic way.

These notes will leave many interesting questions unaddressed. For example, we shall not consider the alternative of postponing expenses in order to learn more on the likelihood of the calamity or on its real effects (see Ulph and Ulph, 1997, for pioneering work in this area), we shall not evaluate the optimal time for the adoption of mitigation and adaptation policies, and we shall not examine the problems of coordination of actions of many agents.

The organization of this presentation is the following. First of all, we discuss very simple projects of mitigation and adaptation and evaluate the key determinants of their net present value, particularly the probabilities of calamity in the long run and the hazard rate. Secondly, we present the fundamental trade-offs of economies in terms of growth and show

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[4] See Weitzman (1998) for a less dramatic case, but one that highlights the importance of the rate of discount. Notice that h works as a discount rate. Rather than a risk premium, h indicates the certainty of convergence to a bad state of nature.

[5] The case of more than one agent (externalities) and the need to model conjectures on the actions of others opens very interesting dimensions that are not explored in these notes.

how the long-run capital stock and standards of living can be affected. In Section 3, we consider the case of choice of technology and show that the economy will probably reduce the use of pollution technologies but, as will be shown in Section 4, long-run behavior may include fluctuations between concern and neglect on climate change issues. In Section 4 we shall also briefly discuss the case of non-optimal growth in order to gain insights on what could happen when outcomes depend on more myopic agents. We will conclude with a summary of basic results in Section 5.

2. Mitigation and Adaptation Projects Evaluation

For the sake of illustration and to make an initial exploration of results, let us first discuss first a very simple case of mitigation and adaptation projects. We follow Kamien and Schwartz (1971a and 1971b) and define $(1 - \psi)$ as the conditional probability that the calamity will happen at time t if it has not happened yet.

Let us assume that the evolution of conditional probabilities is given by:

$$(1) \qquad d\psi /dt = h(\varepsilon - \psi).$$

The constant h is the hazard rate. It indicates the speed of change of the conditional probability of the calamity. Kamien and Schwartz also assume that ε is equal to one, since they are inspired by the literature on failure of machines or other devices.[6] In our case, that would mean that the calamity is unavoidable, since from (1), by integration it is possible to obtain:

$$(2) \quad \psi (t) = (\psi_0 - \varepsilon)e^{-ht} + \varepsilon \, ;$$

consequently, $\psi (t) \rightarrow \varepsilon$ as $t \rightarrow \infty$.

We shall relax that assumption by admitting that the residual probability, given by ε, can be less than one. That amounts to saying that even in the far future, mankind can exist with some positive probability since the planet has the capacity to heal from the wounds inflicted wounds.

We shall discuss an infinite horizon case, and the expected discounted utility will be written as:

[6] In their paper on threat of entry (1971a) they assume that the residual probability is one. But that means that entry is unavoidable.

$$(3) \quad \int [u\,(1-\psi)+\psi B]\;\; e^{-rt}\, dt\,.$$

Here r is the discount rate, and u is the utility level when a calamity does not happen; in the worst event, the utility is reduced to some constant B. At every moment, the agent revises the probabilities, given that the event has not happened yet, and recalculates the expected utility; if the calamity has not happened yet he obtains u, but if it has happened, he obtains a reduced level of utility for the entire future. Then (3) is obtained by adding up all discounted expected utilities.

To begin with a simple case, take u and B as constant. Also suppose that mitigation expenses, I, could have a positive effect by reducing h and/or ε, i.e. $h'(I) < 0$ and/or $\varepsilon'(I) < 0$. So, mitigation expenses contribute to postponing the calamity or to reducing its residual probability. Instead, we shall assume that the effects of adaptation expenses are limited to increasing the utility when the calamity actually happens: $B'(I) > 0$.[7]

Then, by integration of (3) we obtain:

$$(4) \quad \int\; [u\,(1-\psi)+\psi B]\;\; e^{-rt}\, dt = (B-u)[\,(\psi_0-\varepsilon)/(h+r) + \varepsilon/\,r] + u/r$$

For the sake of simplicity, and as an example, let us assume that a project of investment of a small size, let us say $\Delta I = 1$ unit, is being evaluated, that the marginal utility of consumption is one, i.e. $u'(C_0) = 1$, and that the amount invested does not change u.

Under that very simple setup, the marginal net present value of a mitigation project that changes the residual probability $MgNPVmrp$ will be given by:

$$(5) \quad MgNPVmrp = \; -1 + (B-u)\;\varepsilon'(I)h\,/r(h+r).$$

Instead, if the expenses modify h the net present value $MgNPVmh$ will become:

$$(6) \quad MgNPVmh = \; -1 - \; (B-u)\;(\psi_0-\varepsilon)h'(I)/(h+r)^2.$$

Finally, for the case of an adaptation project the marginal net present value will be:

$$(7) \quad MgNPVad = \; -1 + B'(I)\;[\,(\psi_0-\varepsilon)/(h+r) + \varepsilon/\,r].$$

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[7] Though mitigation and adaptation expenses are not necessarily substitutes, they are treated here as independent—see Ingham et al. (2005b) for a discussion of the case of complementarities between mitigation and adaptation.

What are the key determinants of the decision to undertake the additional project (or not) in each one of these cases?[8] As expected, all projects depend on the rate of discount *r*: the higher the opportunity cost for capital, the lower the net present value of additional expenses. But now we see that all projects depend also on the estimated effectiveness of expenses in reducing the residual probability, on the hazard rate and on utility after the calamity.

Equation (5) shows the relevance of the effects of mitigation on the residual probability, for when $\varepsilon'(I) = 0$ the project will be rejected. Instead, mitigation expenses for reducing the hazard rate and adaptation expenses depend heavily on the difference between the perceived current probability of the calamity and its long-run level, given by $(\psi_0 - \varepsilon)$: if they are not perceived as different, then the mitigation project will be rejected and the acceptance of the adaptation project will at least be put in doubt.

One lesson is that the perception of how expenses influence probabilities plays a significant role, and in turn, probabilities have an important subjective component, which depends on the beliefs of agents as well as on scientific information, its quantity and quality.

Also, the net present value of mitigation projects is sensitive to the expected difference in standards of living. The belief that utility will not change much if there is an environmental shock will reduce the net present value of projects.

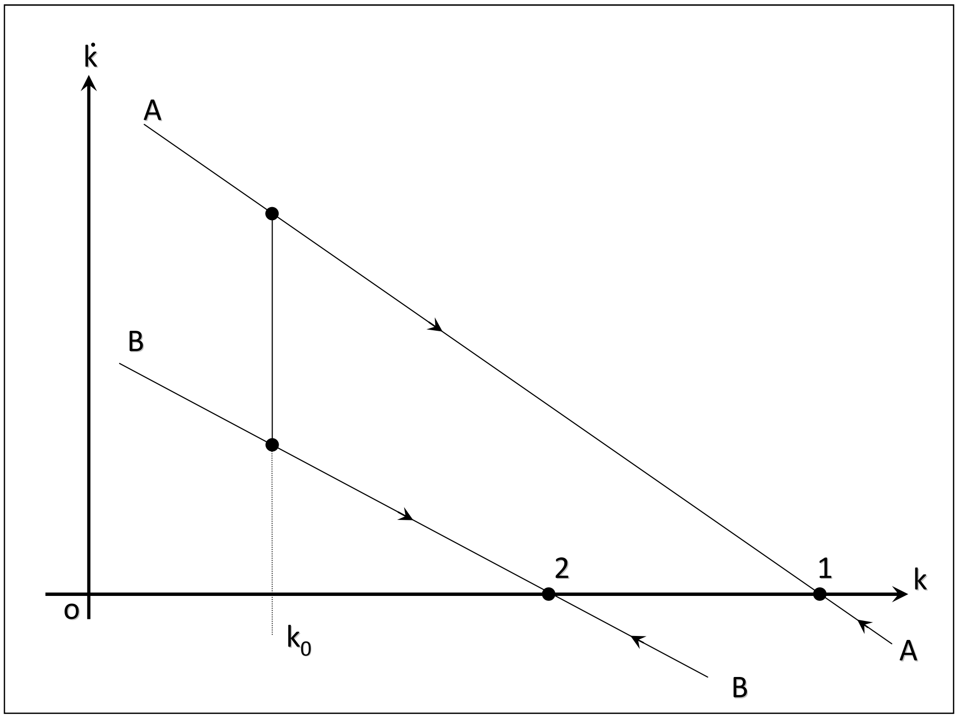
Up to now, the analysis was limited to an individual project and we did not consider the case of macroeconomic choices and to the optimal accumulation of capital. That is the subject of the next section.

3. An Optimal Growth Problem with Adaptation and Mitigation Expenses

As noted above, said, the simplified presentation of the previous section did not take into account the impact of additional expenses in mitigation and in adaptation on the discounted flows of utility in a macroeconomic model. Figure 1 helps to illustrate some of the questions that the optimal growth problem poses.

[8] Inaction is a possible optimal action when the marginal gains are low compared to the initial investment; see Stokey (2008).

Figure 1. Climate Change, Long-Run Capital and Speed of Convergence



On the axes we have the stock of capital *k* and net investments, *dk/dt*. The line AA represents the saddle path of the original economy, without climate change threats and without compensatory policies (mitigation and adaptation). Let us assume that the economy is at k_0 and that it is moving towards a steady state indicated by **1**.

It can be conjectured that the emergence of the threat and preventive expenses will have consequences for both the long-run stock of capital and the speed of convergence to the steady state. Those presumptions are in fact confirmed by our general model in the following sections.

The probability of the calamity and the necessary mitigation or adaptation expenses can change the general picture, since they will modify: i) only the long-run level of the capital stock, ii) only the speed of convergence to it, or iii) both, as in the new saddle path represented by BB. In the case of this portion of the figure, it is assumed that the long-run stock is smaller since **2** is located to the left (that is, the expected shock plus the expenses oblige the economy to produce less), and also that the economy grows at a slower pace, since BB is below AA. Of course, the smaller long-run capital stock will imply a reduced permanent standard of living.

There are other many alternative cases and scenarios; in some cases mitigation and adaptation expenses may have a positive impact—for example, through new scientific discoveries. We are not going to consider the case of gains due to research and development fostered by the threat, although they could have positive externalities on growth. Instead we shall address the case of substitution of technologies in Section 3.

So to discuss a more general case, let us assume that the expected utility is given by:

$$(8) \quad \int [u(C)(1-\psi)+\psi B(Q)]\; e^{-rt}\, dt$$

In this case $u(C)$ is a standard neoclassical utility function that depends on consumption, C. When life is perturbed by the calamity, utility is changed to $B(Q)$; while B was a constant in previous section, now it is considered a function (with positive derivative, i.e., $B'(Q) > 0$) of accumulated adaptation expenses, represented by Q.

Notice that:

- If it were assumed that "extinction is forever", utility would be modified to $B(Q) \equiv 0$ for all $t > t^*$. Extinction forever could be then a special case of Dasgupta and Heal's (1980) dilemma, in which artificial capital cannot be used as a substitute for natural capital.
- When $B = 0$, the discount factor is $(1 - \psi)$ $e\text{-}rt$. Then the implicit discount rate will not be decreasing, and therefore the optimal path does not require commitments due to the existence of dynamic inconsistencies; see Laibson (1997).
- It is also admissible that $k(t^*)$ could be relevant for welfare in the second stage, after the calamity. In that case, it will be necessary to explore complementarities or substitutabilities at the level of $B(Q,k)$, which are not considered in these initial notes.

When the outcome is not extinction, then the utility B will be result of the accumulation of adaptation expenses a:

$$(9) \qquad dQ/dt = a.$$

Total expenses in consumption, mitigation and adaptation must respect the budget condition:

$$(10) \quad F(k) = C + m + a + dk/dt + \delta k.$$

Here $F(k)$ is a neoclassical production function, with the habitual regularity properties. Parameter δ is the depreciation rate of artificial capital, and m stands for mitigation expenses.

We can argue that the role of mitigation expenses is to reduce the change of the calamity, and it is manifested through a function $b(m)$ (such that $b'(m) \geq 0$) in the law of evolution of probabilities. This now can be given the form:

$$(11) \quad d\psi /dt = h(F(k) - b(m))\, z(\varepsilon - \psi)$$

With regards to this optimization program we can state:

1. There is an asymmetry in this treatment between mitigation and adaptation expenses: it is assumed in this special case that mitigation expenses are not accumulative.
2. As in the previous section, we assume that $h(F(k) - b(m)) > 0$,[9] so that ψ tends to ε, which is not necessarily one; thus, the calamity is not certain even in the far future, but it is possible.
3. The function $(\varepsilon - \psi)$ is replaced by a more general expression, $z(\varepsilon - \psi)$, where z is non-negative, $z(0)= 0$, and $z'(\varepsilon - \psi) > 0$; therefore it is still possible to find a steady state for ψ.[10]
4. Though in the long run the probability of the calamity could be one, the representative agent estimates that probability as ε. Gaps between the levels of the actual and the perceived residual probabilities might create a difference between objective and subjective optimal policies. Quite the same can be said about functions h and b; a pessimistic view will consider that b is zero, and therefore mitigation expenses will be useless.

[9] It could be argued that the discount rate becomes endogenous. The distant future could be discounted at higher rates when the certainty of the calamity is increasing. That will increasingly favor current generations over future generations; see Weitzman (1998).
[10] When $\varepsilon = 1$ even the concept of steady state is more elusive, since it implies that calamity is certain. In this model we prefer to leave open that the doomsday is only possible with positive probability but less than one.

As we shall discuss later, this teaches that the communication of the scientific certainties (or uncertainties) could influence the intertemporal policy; however, in the example below, they are not relevant for the capital stock in the long run.

5. All functions are assumed to fulfill the concavity conditions that guarantee the optimality of the path.

The Hamiltonian $H(t)$ of the problem is now:

$$(12) \quad H(t) = [u(C)(1-\psi) + \psi B(Q)]\}e^{-rt} + \lambda[F(k) - C - m - a - \delta k] + \\ + \mu [h(F(k) - b(m))\, z(\varepsilon - \psi)] + \rho\, a$$

In this expression, ρ, λ and μ are the adjoint variables for (9), (10) and (11) respectively. And the necessary conditions for an optimal solution are given by:[11]

$$(13) \quad u_C (1-\psi)\, e^{-rt} - \lambda = 0,$$

$$(14) \quad -\lambda - \mu h'(F(k) - b(m)) b'(m)\, z(\varepsilon - \psi) \leq 0,$$

$$(15) \quad -\lambda + \rho \leq 0,$$

$$(16) \quad \psi B'(Q) e^{-rt} = -d\rho/dt,$$

$$(17) \quad \lambda[F'(k) - \delta] + \mu\, h'(F(k) - b(m)) F'(k)\, z(\varepsilon - \psi) = -d\lambda/dt,$$

$$(18) \quad [-u(C) + B(Q)]\}e^{-rt} - \mu\, h(F(k) - b(m))\, z'(\varepsilon - \psi) = -d\mu/dt.$$

The main results of these equations are the following:

- In the steady state, the economy does not expend resources in mitigation. This is obtained from (14), taking into account that $z = 0$ at the steady state, and using a Kuhn-Tucker condition for m, so that: $-\lambda m = 0$.
- But if ε is a function of m the long-run optimal expenses in mitigation may be positive. That can be seen by replacing the expression (11) with $d\psi/dt = h\, z(\varepsilon(m) - \psi)$ and computing again the necessary conditions for an optimum. In this case, though, there is not reversibility in the

[11] And the transversality conditions: $\lim_{t \to \infty} \lambda = \lim_{t \to \infty} \mu = \lim_{t \to \infty} \rho = 0.$

model and the effects of the calamity (let us say, "extinction") will be forever, the agent has the chance of reducing the residual probabilities.

- Also from (14), $\mu < 0$, the shadow price of ψ is negative, since it is a stock that reduces expected utility. And from (17) this will imply that the rate of accumulation of capital will be smaller—as shown in Figure 1, for example, BB will be under AA.
- It is not possible to rule out positive adaptation expenses at the steady state, since (15) can be solved by taking $\lambda = \rho$.
- There is a fundamental trade-off between consumption at time T and the probability of increasing quality of life if the calamity does happen. This is seen noting that from (15) and (16): $u_C\ (1 - \psi)\ e^{-rT} = \int_T \psi B'(Q) e^{-rt}$. This presents a trade-off between: 1) the additional utility obtained from increasing consumption at time t, or 2) using that unit of the good to extend the "quality" of life for the period following the calamity.[12]
- Given a long-run level of ε, mitigation and adaptation expenses will have no influence on the long-run capital stock when $F(k)$ is limited to being a determinant of h[13] but not of the residual probability. To prove this, take the time derivative of (13) and use (10), remembering that $z = 0$ in the steady state. This produces the familiar condition for the long-run capital stock: $F'(k^*) = r + \delta$. The corollary is that the economy will not be producing more goods, and mitigation and adaptation expenses will be subtracted from consumption: in the absence of technological advances, the quality of life will have to be lower! In terms of Figure 1, points **1** and **2** will have the same location, but the slope of AA will be steeper than the slope of BB.
- However, if is the residual probability that is a function, i.e., $\varepsilon\ [F(k) - b(m)]$ with $\varepsilon' > 0$, then the long-run capital k^* will be lower: $F'(k^*) > r + \delta$. If production is a source of risk, and if that risk manifests itself at

[12] It has the flavor of the trade-off noted by Loury (1978) between present gains of utility and extending life.
[13] Mitigation expenses can have cumulative effect M, i.e., $b(M)$ with $dM/dt = m$ instead of $b(m)$. But the long-run capital is not modified in that case.

the level of the residual probability, then the stock of capital will be reduced in the long run.[14] In terms of Figure 1, point **2** will be located to the left of point **1**, as shown there.

An important corollary is therefore that perceptions of the impact of mitigation expenses are important. Stating that those expenses modify the velocity of convergence to the calamity is not the same as stating they reduce its residual probability.

3. Choice of Technology

Let us now consider that the economy can select a technology of production that does not pollute, for example, one that does not contribute to GHG emissions. In the absence of a threat, capital will be allocated so that marginal productivity is the same for both technologies, but total capital will not be reduced. That is not necessarily the result under threat.

To see that, define $G[(1-\gamma)k]$ as the production function for the "clean" technology when a proportion $(1-\gamma)$ of total capital is employed in it. The rest of the capital stock γk is used in the more polluting traditional technology, represented by $F(k\gamma)$. We assume that the switch between technologies is not costly (see Acemoglu et al., 2010, for a discussion of choice of technology and optimal intervention with endogenous technical change under different costs of switching). In the case of their paper, long-run growth is not affected when switching costs are zero; in the case of this paper, it depends on which element of the law of evolution of probability is affected. Also in our case, the problem of the design of intervention policies is assumed solved by our social planner.

So, let us consider first the case in which the choice of allocation of capital between the technologies impacts the hazard rate.

The Hamiltonian is:[15]

$$(19) \quad H(t) = u(C)(1-\psi)e^{-rt} + \lambda[F(k\gamma) + G[(1-\gamma)k] - C - \delta k] + \\ + \mu\,[h(F(k\gamma)\,z(\varepsilon-\psi)]$$

Then the necessary conditions are:

[14] Consequently, this is also the case when the residual probability is a function of C.

[15] Inaction could be the result of adjustment costs, for example of the form $c(d\gamma/dt)^2$.

$$(20) \quad u_C\,(1-\psi)\,e^{-rt} - \lambda = 0$$

$$(21) \quad \lambda\{[F'(k\gamma) - G'[(1-\gamma)k]]k\} + \mu h'k\,z(\varepsilon - \psi) = 0$$

$$(22) \quad \lambda\{F'(k\gamma)\,\gamma + G'[(1-\gamma)k](1-\gamma) - \delta\} + \mu\, h'F'\gamma z(\varepsilon - \psi) = -$$

$$- d\lambda/dt$$

$$(23)\ [-u(C) + B]\}e^{-rt} - \mu\, h(F(k\gamma)\, z'(\varepsilon - \psi) = -d\mu/dt$$

It can be shown that at the steady state:

$$F'(k\gamma) = G'[(1-\gamma)k] = r + \delta,$$

The paradox is that, even if the transition could possibly require less pollution, once the steady state is reached the polluting technology will not be used on a smaller scale![16] These results confirm what we have seen in the previous section. The rationale is that at the steady state there are no additional gains in terms of reduction of the residual probability of a calamity.

So, we now explore the alternative case, when pollution is included in the residual probability. Then the Hamiltonian becomes:

$$(24) \quad H(t) = u(C)\,(1-\psi)e^{-rt} + \lambda[F[k\gamma] + G[(1-\gamma)k] - C - \delta k] + \\ + \mu\,[h\, z(\varepsilon(F(k\gamma) - \psi)]$$

Then the necessary conditions are:

$$(25) \quad u_C\,(1-\psi)\,e^{-rt} - \lambda = 0$$

$$(26) \quad \lambda\{[F'(k\gamma) - G'[(1-\gamma)k]]k\} + \mu hk\, z'(\varepsilon - \psi)\,\varepsilon' F' = 0$$

$$(27) \quad \lambda\{F'(k\gamma)\,\gamma + G'[(1-\gamma)k](1-\gamma) - \delta\} + \mu\, hF'\gamma z'(\varepsilon - \psi)\,\varepsilon' = -d\lambda/dt$$

$$(28)\ [-u(C) + B]\}e^{-rt} - \mu\, h\, z'(\varepsilon - \psi) = -d\mu/dt$$

Then at the steady state:

$$(29) \quad F' > G' = r + \delta.$$

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[16] When F and G fulfill Inada's conditions, production will be positive with both technologies. For example when $G = v\,F$, with $0 < v < 1$, and $G = Ak^{\alpha}$ then $\gamma = 1/(1 + v^{1/\alpha})$.

Therefore, the optimal long-run capital stock will be lower when the choice of one of the technologies contributes to increasing the long-run probability of a calamity. The corollary is that the economy will produce a smaller quantity of goods in the long run, and the standard of living will have to be also reduced! But it should again be noted that we are not considering the chances of gains provided by research.

4. The Dynamic Path

Regular solutions for the dynamic optimization problem should give saddle-path dynamics, the kind of solution shown in Figure 1. It is not necessary, however, to have a saddle-path case for compatibility with optimal control problems. Instead, it is sufficient to have bounded trajectories that verify the transversality conditions; see, for example, Benhabib and Nishimura (1983). There is still work to do in the case of our model, but it is possible to suggest that the saddle path property will not necessarily be the only kind of dynamic solution.

To see that, let us now consider now the following specialized case:

$$Max \int [u(C)\,(1-\varepsilon(k))+\varepsilon(k)\,B]\ e^{-rt}\,dt$$

Here we assume that $\varepsilon'(k) > 0$, that is, the stock of capital increases the chance of a calamity, but in this case it does so directly and instantaneously and not through the law of evolution of conditional probabilities given by (11). The problem will be subject to (10) without adaptation and mitigation expenses, i.e.,

$$F(k) = C + dk/dt + \delta k.$$

In this case, the long-run stock of capital will be lower, and given by

$$F'(k^*) = (r + \delta)/(1-\varepsilon(k^*)) > r + \delta.$$

First results indicate that the steady state could be a saddle point or even exhibit Hopf limit cycles depending on the values of B and $\varepsilon'(k^*)$. The reason is that the trace of the system takes the value of zero in a neighborhood of the steady state (while the determinant is positive).

This implies that the society could be following a line like AA or BB or an optimal cycle around the long-run steady state **2** in Figure 1. In the second case, periods of concern about climate change could be followed by periods of neglect and exuberance in consumption!

However, let us now abandon the normative model and consider the interface between growth and the probability of a calamity from the perspective of economic forces that might push the economies out of their optimal or recommendable paths. That could give a hint of what will be observed in the case of models based on agents; it is difficult a task for our simplified construction, since models based on agents have a high sensitivity to their architecture. But to get at least some insight, we can postulate this simplified version of the dynamic system:

$$d\psi /dt = h(k)\,(\varepsilon - \psi)$$

$$dk/dt = g\,[\,F'(k)\,(1- \psi) - r - \delta]$$

The capital stock modifies the hazard rate but not the residual probability, and the capital stock adjustment responds positively to the difference between expected marginal product and cost of capital. Then, for the linear approximation the trace will be $-h + g\,F'' < 0$, and the determinant $-gh\,F'' > 0$. Therefore the steady state is stable, but it is not a saddle point.

Something similar happens when the dynamics are governed by:

$$d\psi /dt = h\,(\varepsilon(k) - \psi)$$

$$dk/dt = g\,[\,F'(k)\,(1- \psi) - r - \delta]$$

Notice that in this case, it is the residual probability that depends on the stock of capital (more capital implies greater chances of a calamity). The trace is $-h + g\,F''(1- \varepsilon)$, which is again negative, but the sign of the determinant is defined by $-h\,g\,F''(1- \varepsilon) + hgF'\,\varepsilon'$ and since $\varepsilon' > 0$, it is positive and it is stable, and it is not a saddle point.

So, stability of the steady state prevails in both cases.

5. Concluding Remarks

To summarize some of the main findings of our discussion we can state the following:

- We considered the case of the representative agent of a society that maximizes discounted expected utility and designs an optimal growth program under threat of a climate change calamity.
- We adopted Kamien and Schwartz's strategy and said that at every moment the agent estimates conditional probability of occurrence of the event for every time, given that it has not happened yet (once the event happens it will stay for the following periods, and it is irreversible).
- This representative agent fully internalizes the consequences of pollution and GHG emissions, so we did not address the problems of corrections of externalities between different agents.
- Our results indicate that the vision of how the probability of the calamity evolves, and of the level of the residual probability (that the calamity can happen, even after human correction of emissions), is important for willingness to contribute to climate change expenses in the long run.
- For example, optimal mitigation expenses will be positive in the long run when they impact on the residual probability of the calamity, but they will be zero when they affect only the speed of convergence of probabilities.
- Therefore, the perception of the deep causes of the calamity and of the law of evolution of probabilities influences the optimal allocation of expenses and willingness to contribute.
- As a corollary, the communication of scientific information to society becomes a relevant variable.
- Consistently, the perception that production contributes to increase the speed of convergence to the calamity will not modify long-run capital.
- Instead, if it is presumed that production impacts negatively on the residual probability of the calamity, then long-run capital will be reduced.

- The same optimal response (reduction of the long-run capital stock) is confirmed when the economy has more than one technology available, for example, when there exists a choice between polluting and clean production functions.
- Additionally, adaptation investments call attention to the fundamental trade-off between sacrificing current welfare to increase the standard of living after the calamity occurs.
- The dynamics do not necessarily respond to the saddle-point property, and it is not possible to rule out cyclical behavior. That means that the optimal dynamic behavior might imply fluctuations, and economies might have optimal endogenous cycles of environmental concern and neglect.

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